Technical Comments

Comment on "A New Algorithm for the Computation of the Geodetic Coordinates as a Function of Earth-Centered Earth-Fixed Coordinates"

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LUPASH[1] has presented an algorithm that iteratively solves for the point on the reference geoid below a spatial position point. This algorithm reduces to the solution of a quartic equation in the scaled variable v. An iterative method is presented for estimating the proper solution to the equation. This method is compared to a competitive algorithm that requires the evaluation of a square root within the iterative loop. Only one reference is cited and that is a set of unpublished notes.

The purpose of this Comment is to present some more bibliographical information relative to this algorithm. Details of the alternative algorithms will not be presented.

Baird[2] presented an equivalent algorithm in 1964. This requires the iterative solution of the following quartic equation for the unknown variable z, the Earth polar axis coordinate of the point on the geoid:

$$f(z) = K_1 z^4 + K_2 z_e z^3 + (K_3 + K_4 z_e^2 - R_0^2) z^2 + K_5 z_e z + K_6 z_e^2$$

where the following definitions of the variables not found in Ref. 1 apply:

$$K_1 = -e^4/(1-e^2) \qquad K_5 = 2a^2e^2(1-e^2)$$

$$K_2 = -2e^2 \qquad K_6 = a^2(1-e^2)^2$$

$$K_3 = a^2e^4 \qquad R_0^2 = x_e^2 + y_e^2$$

$$K_4 = -(1-e^2)$$

Given an initial estimate for the unknown z, the standard Newton-Raphson method converges very rapidly. Within the iteration loop approximately seven multiplications, seven additions, and one division are required. Convergence is not a function of the distance from the equator or poles. However, calculation of the altitude from the solution must branch on nearness to the equator. Also, if the trace of the trajectory crosses the equator, the sign of z must agree with the sign of z_e.

The Baird algorithm has been successfully used for many years in a simulation program.[3] This program has simulated rocket trajectories from such diverse places as Kwajalein Island and Churchill, Canada.

A detailed analysis of the merits of the Lupash and Baird algorithms would be required to chose between them, especially since algebraically they rest on the same relationships of an ellipsoid of revolution. The Lupash algorithm works in scaled variables. The Baird algorithm works in dimensional quantities. It definitely requires extended precision, which might be optional for Lupash algorithm.

A different method of solving this problem is given by Gersten.[4] This involves no iterations but does require the evaluation of trigonometric functions.

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References

[1]Lupash, L. O., "A New Algorithm for the Computation of the Geodetic Coordinates as a Function of Earth-Centered Earth-Fixed Corrdinates," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 8, Nov.-Dec. 1985, pp. 787-789.

[2]Baird, R.W., "Cartesian Coordinate to Geodetic Coordinate Conversion," White Sand Missile Range Data Reduction Directorate, June 1, 1964.

[3]Churchyard, J.N., "Three Degree of Freedom Trajectory Program Description," Missile System Div., Atlantic Research Corp., Rept. AR/MSD-127-OOP, July 1968.

[4]Gersten, R., "Geodetic Sub-Latitude and Altitude of a Space Vehicle," *Journal of the Astronautical Sciences*, Vol. III, No. 11, Spring 1961.

Reply by Author to J.N. Churchyard

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DR. Churchyard's comments on Ref. 1 are welcome, since an algorithm that is not available in open literature is mentioned.

I want to make a correction of the number of elementary operations specified in Dr. Churchyard's comments. The Baird algorithm requires practically the same number of elementary operations as the algorithm presented in Ref. 1, within the iterative loop. A minor difference of one operation per iteration can occur if the stopping condition used by the Baird algorithm is the absolute value of the difference between two consecutive values instead of the relative value used in Ref. 1.

With the assistance of Dr. Churchyard, I have had a chance to review briefly Ref. 2. The Baird algorithm presented there has a completely different derivation from the algorithm presented in Ref.1, in spite of the fact that the final quartic equations are similar. The initialization of the iterative process is not clearly specified in Ref. 2 and some work is needed to put the Baird algorithm presented there into a proper form for mechanization.

References

[1]Lupash, L.O., "A New Algorithm for the Computation of the Geodetic Coordinates as a Function of Earth-Centered Earth-Fixed Coordinates,"*Journal of Guidance, Control, and Dynamics*, Vol.8, Nov-Dec. 1985, pp. 787-789.

[2]Baird, R.W., "Cartesian Coordinate to Geodetic Coordinate Conversions," White Sands Missile Range Data Reduction Directorate, June 1, 1964.

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